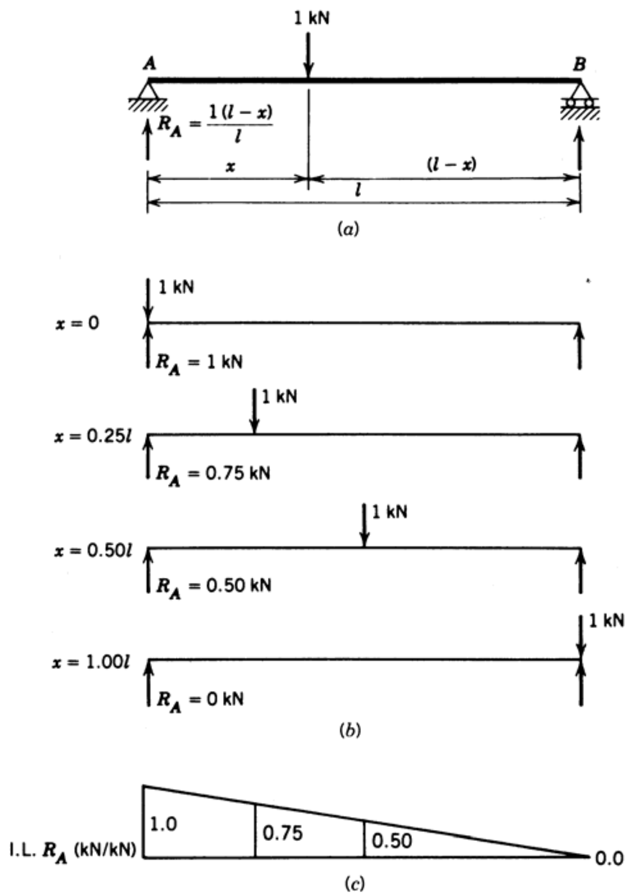
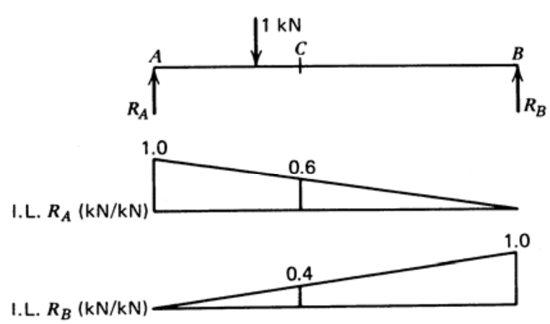
Influence Lines

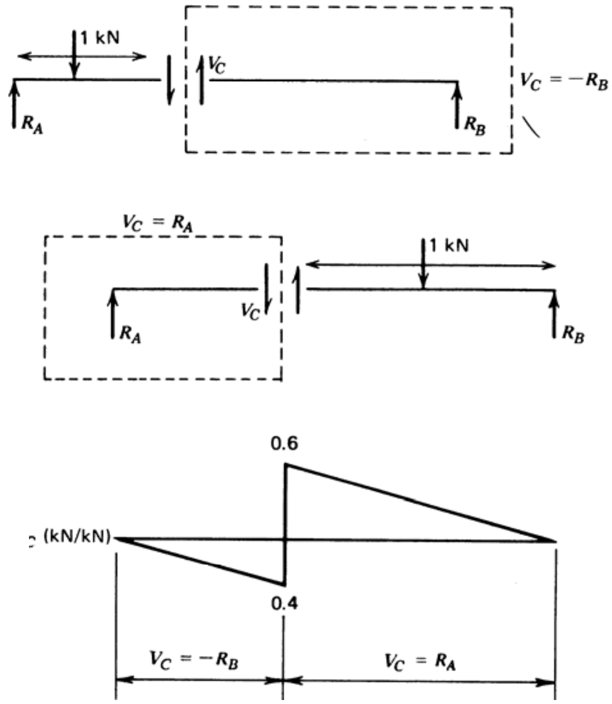
Definition: An influence line shows graphically how the movement of a **unit load** across a structure influences a force effect (reaction, axial force, shear, or bending moment) at **one point** in the structure. **IT IS NOT** a shear force or bending moment diagram. A bending moment diagram, for example, represents graphically the variation of bending moment at all points in a structure under one set of loads.

<u>Influence line for reaction of a simply supported beam</u>

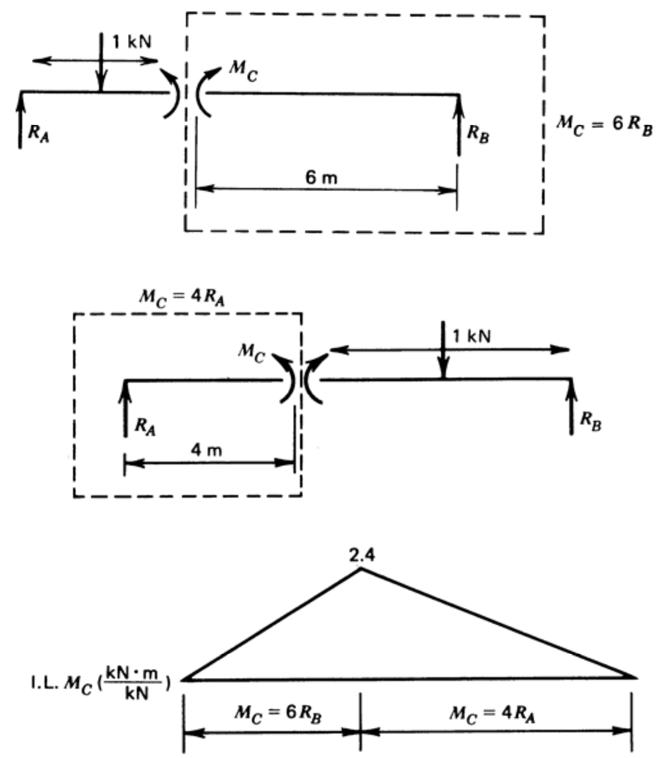




Influence line for shear in a simply supported beam (equilibrium method)

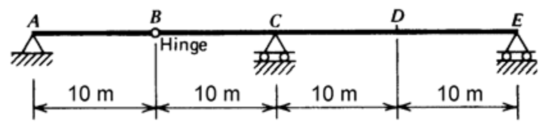


Influence line for moment at a point in simply supported beam



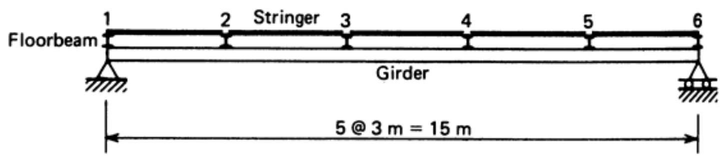
Example1

Construct the influence lines for the shear at the hinge at B and for the moment at D for the structure shown below.



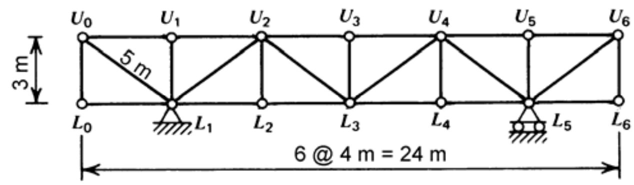
Example 2

Draw the influence lines for the shear in panel 2-3 and the moment at point 5 for the girder shown below. The unit load is applied to the stringers.



Example 3

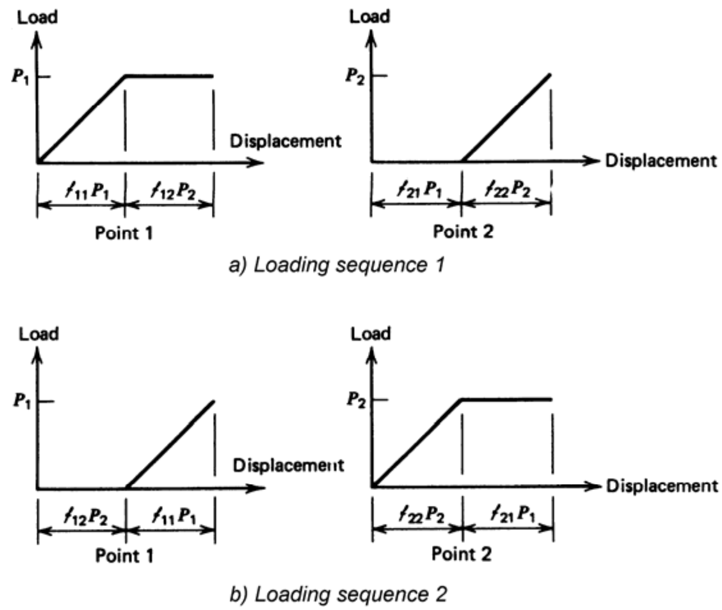
Construct the influence lines for the forces in members U_1L_1, U_2L_3, and U_3U_4.



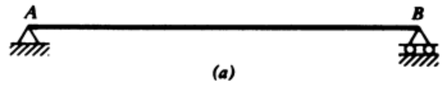
Note: A truss can receive loads only at joints. When the unit load is between the joints of a chord member it will enter the truss through adjacent joints by shear and flexural action of the truss member.

Influence Lines by Virtual Work (Müller-Breslau Pinciple)

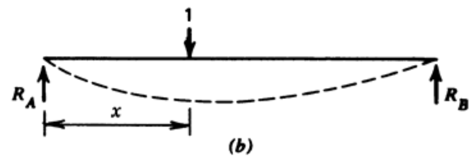
Although the method of equilibrium is very useful to understand the concept of influence lines, it is rather time consuming. Once we understand the concept influence lines, we seek a more effective method to obtain influence lines. As it turns out, Betti's law (***<u>10% bonus mark on your midterm exam for the first one who can tell me what is Betti's law!***</u>) can be used to develop a very useful concept for the construction of influence lines.



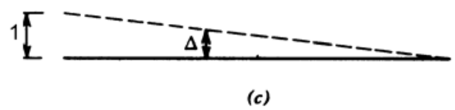
<u>Recall:</u> f_{ij} is a flexibility term defined as the displacement at i due to a unit load at j. P_1 and P_2 are loads applied at two different locations on an elastic body.



Given structure



System 1: structure in equilibrium under unit load.



System 2: structure subjected to unit displacement corresponding to R_A.

From Betti's law, the above figure can be expressed as follows:

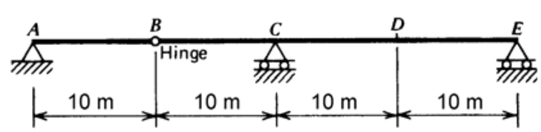
$$R_A \times 1 = 1 \times \Delta$$

which indicates that the reaction at A is equal to the deflection of the structure due to a unit deformation at A in the direction of the reaction force at A. This applies for any location of the unit load. Therefore, the influence line for the reaction force at A corresponds to the deflected shape of the structure when displaced by a unit at A in the direction of the reaction force.

Similarly, the influence line for any force effect is given by the deflection curve that results when the restraint corresponding to that force effect is removed and a unit displacement is introduced in its place. This is known as Müller Breslau's principle

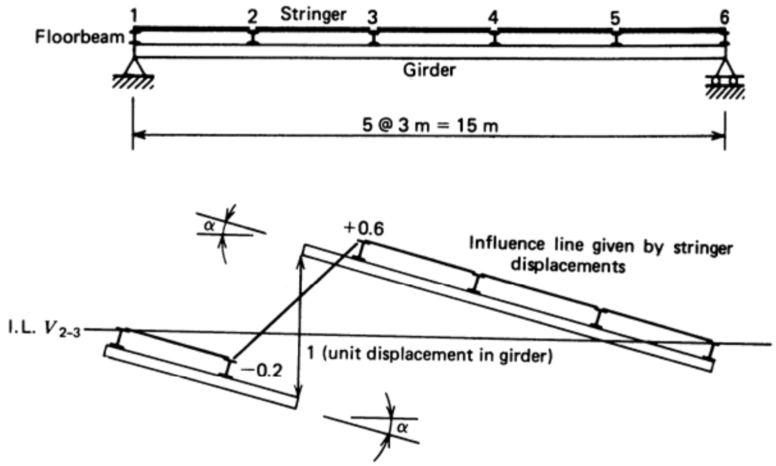
<u>Example 4</u>

Repeat Example 1 using Müller Breslau's principle.



Example 5

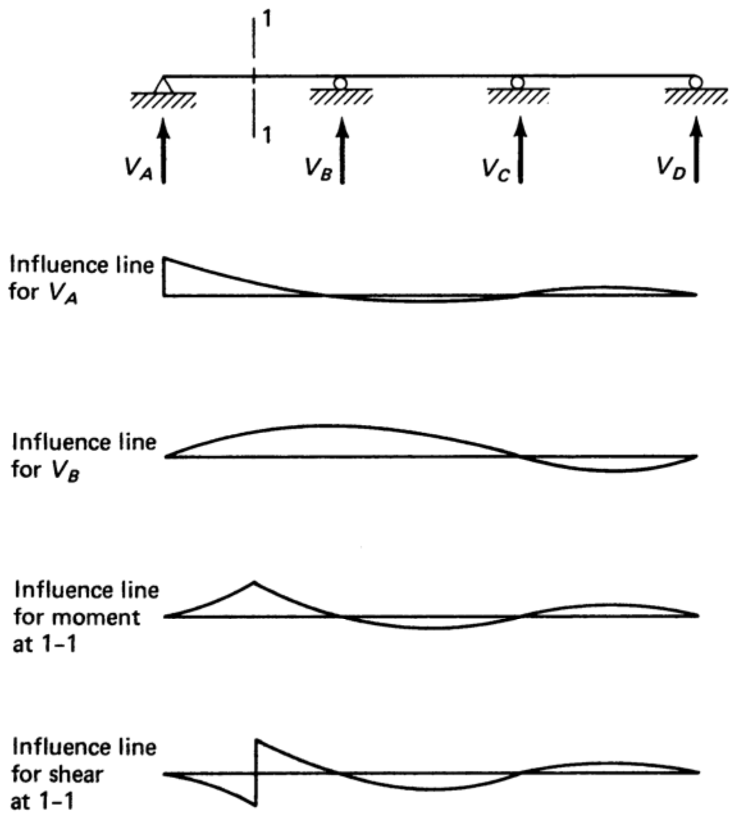
Repeat Example 2 using Müller Breslau's principle.



Influence lines for statically indeterminate structures

Although the same principles presented above for statically determinate structures apply for statically indeterminate structures, the calculations required to obtain the influence lines for a statically indeterminate structure can be quite tedious. Müller Breslau's principle can be used to obtained a *qualitative* influence line.

When we used Müller Breslau's principle for statically determinate structures, as a force effect is released to obtain the corresponding influence line, the statically determine structure become unstable. The structure therefore deforms as a series of rigid bars (because no force can be applied to a statically unstable structure, the parts of the structure remain undeformed). This is not the case, however, with statically indeterminate structure. The release of a force effect would, at best, render the structure statically determinate. The application of the unit displacement in the direction of the force effect therefore causes deformations in the elements of the structure. The following example illustrates the application of Müller Breslau's principle for a multispan, continuous, beam.

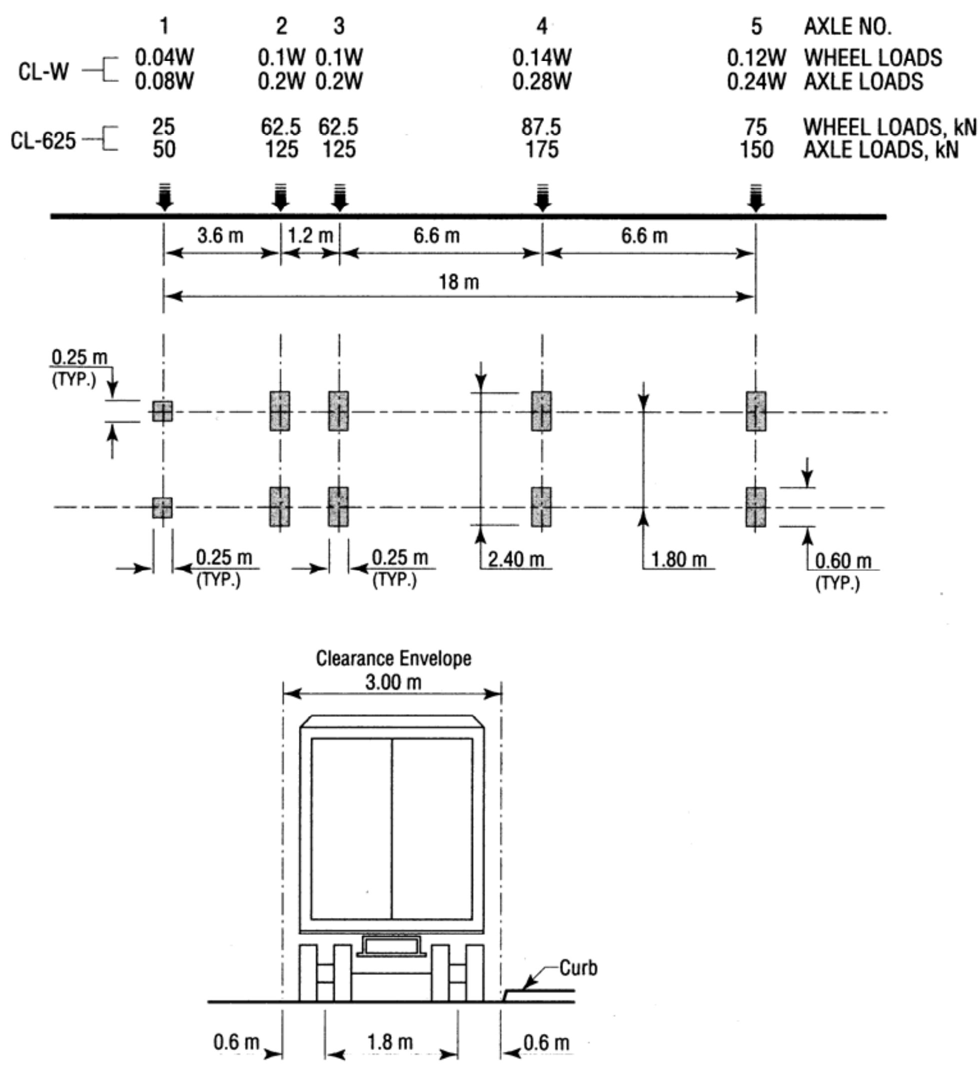


Other example of statically indeterminate structure

Although it is easy to draw the influence line qualitatively for any force effect, the calculation of the ordinate values for an influence line of a statically indeterminate structure is tedious. A common approach for statically indeterminate structure is to use a structural analysis software such as S-FRAME and walk a unit load along the structure. The structure is analyzed for each position of the unit load. All the force effects at all locations in the structure are obtained in the process. The influence line for any force effect obtained in the analysis can then be obtained. Many commercial software accomplish this task automatically, without intervention of the user other than asking for an influence line to be generated.

Moving Loads

Figure 3.8.3.1 CL-W Truck



The truck load illustrated in the figure above is the Canadian Legal truck, designated as CL-W, where W is the total weight of the truck in kilonewtons. The legal traffic loads vary from one province to another across Canada. Even within a province, traffic conditions may vary from one locality to another (e.g. roads in Northern Alberta see heavier loads than most other roads because of the oil industry activities). Therefore, CSA-S6 specifies a certain minimum standard for the highways that carry inter-provincial traffic, and allows flexibility of selecting a load level suitable for other roads and highways. The specified loading can adopt an appropriate level of loading at the discretion of the provincial authorities.

The recommended traffic load possesses the following characteristics:

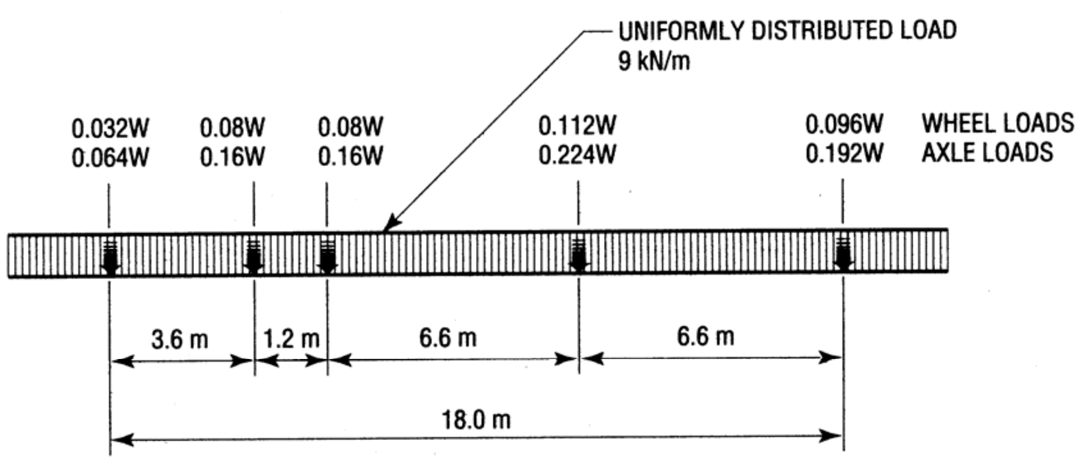
(a) models heavy wheel loads;
(b) models heavy axle loads in a design lane;
(c) consists of one heavy vehicle in a design lane (figure above);
(d) allows for multiple presence of vehicles in a design lane (figure below); and
(e) allows for the simultaneous presence of vehicles or axle loads in more than one design lane.

The CL-W Lane load consists of a lighter design truck, applied in combination with other vehicles, represented by a distributed load. This load governs for the design of long span bridges whereas the heavier truck alone governs the design of shorter span bridges.

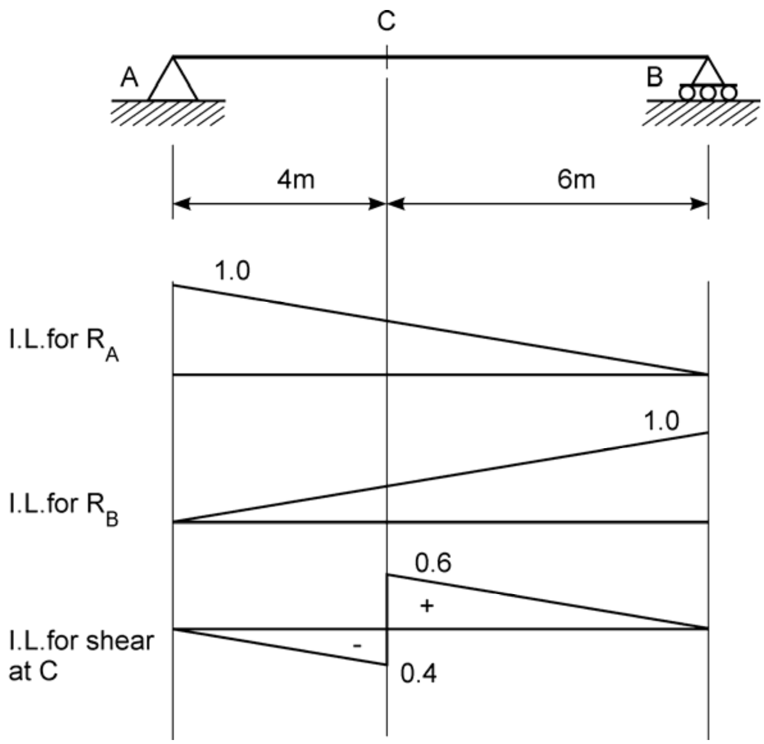
The load magnitude, W, adopted in Canada is 625 kN, although other magnitudes of W are allowed. It should be noted, however, that the code allows for different magnitudes of W. The current load and resistance factors are calibrated for a 625 kN truck.

Read sections C3.8.1 to C3.8.3 of the Commentary to CSA-S6-00 for detailed information about the derivation of the CL-W truck load.

3.8.3.2 CL-W Lane Load



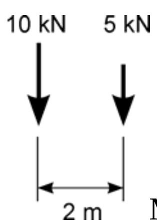
Influence lines are used to determine the load effect in a structure subjected to a point load, a series of point loads, or a uniformly distributed load. For a single point load, the load effect is simply obtained by multiplying the influence value by the magnitude of the concentrated load. For multiple point loads, the load effect is obtained by taking the sum of the product of the influence ordinates by the corresponding concentrated load. For uniformly distributed loads, the force effect is obtained by multiplying the magnitude of the distributed load by the area under the influence line.



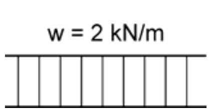
Example 6

Calculate the maximum shear at C for the beam above and the following loadings:

Maximum shear = 0.6 kN/kN x 10 kN = 6 kN



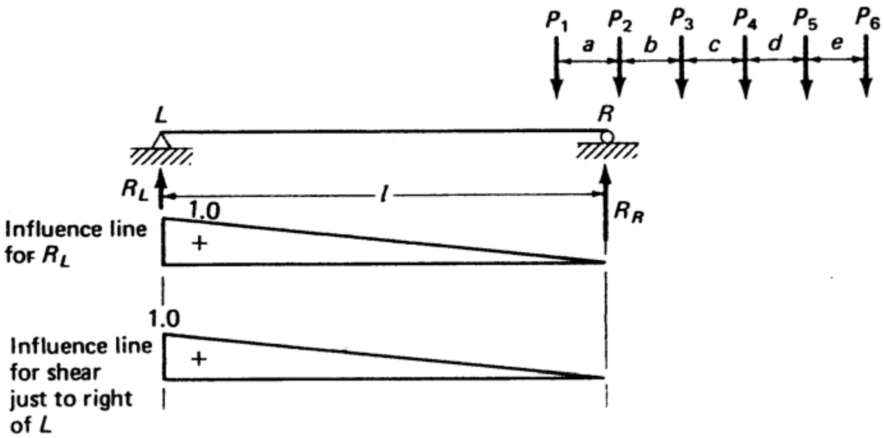
Maximum shear = 0.6 kN/kN x 10 kN + 0.4 kN/kN x 5 kN = 8 kN



Maximum positive shear = (0.6 kN/kN x 6 m)/2 x 2 kN/m = 3.6 kN
Maximum negative shear = (-0.4 kN/kN x 4 m)/2 x 2 kN/m = -1.6 kN

Maximum Reaction (or end shear) in a Beam Supporting Moving Concentrated Loads

If a simple beam is loaded with a series of moving concentrated loads, the maximum shear occurs at the supports. Which position of the loads will cause the greatest reaction (end shear)?



The maximum reaction can be determined by trial and error; move every load P_i over the support, one at a time and determine which one will give the largest reaction (not as tedious as it may sound).

Alternatively, one can the change in reaction as each load passes off the span. As P_1 passes off the span, the shear change is

$$dV = \frac{\sum P\ a}{L} - P_1$$

where $\sum P$ is the sum of the loads remaining on the beam. As P_2 passes off the span and P_3 moves over to the left support, the shear change is

$$dV = \frac{\sum P\ b}{L} - P_2$$

Example 7 (*in class example*)

Maximum Shear at Interior Points of Beams Supporting Moving Concentrated Loads

The maximum interior shear can be determined by a method closely related to the method used for end shear or reaction. Using the trial and error procedure, one tries to place as many loads on the positive portion of the influence line as possible and as few on the negative portion of the influence line. Once again, the method is not as tedious as it may sound.

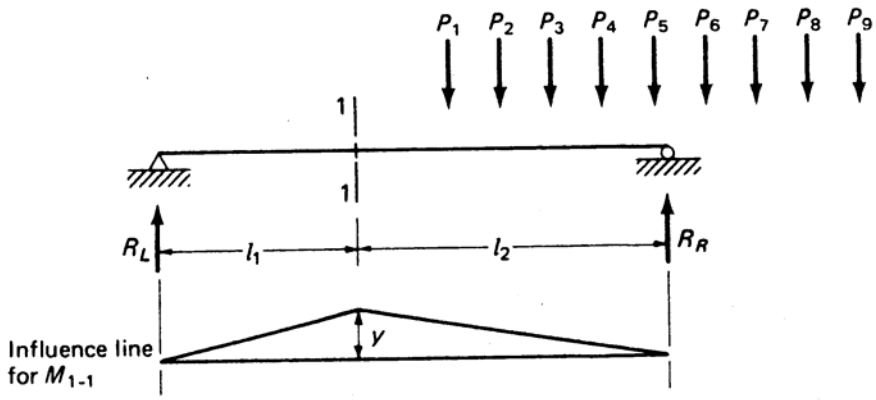
Considering the change in shear as each load passes across the section at which we are seeking the maximum shear force, one can observe that as the loads are moved one after another to the section the shear change equals the increase in the left reaction due to the movement of the loads to the left, plus the increase in the left reaction due to any additional loads that have come onto the span from the right, less the load that has just passed over the section. If the sum of these values is positive, the shear has increased. The first load that in moving past the section cause a decrease is the one that will cause absolute maximum shear and the computations are made with that load at the section.

Example 8 (*in class example*)

For the previous example, determine the maximum shear at a distance 3 m from the left support.

Maximum Moment at a Point in a Beam Supporting Concentrated Live Loads

A study of the moment change as each load moves up to and past the section provides a method of quickly obtaining the exact maximum moment. Such a study demonstrates that the absolute maximum moment at any point in a beam due to a moving series of concentrated loads occurs when the average load to the left of the point is equal to the average load to the right of the point.



This translates into the following for the example shown in the figure above:

$$\frac{\text{total load to left}}{l_1} = \frac{\text{total load to right}}{l_2}$$

Example 9 (*in class example*)